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Similarly, $AB+BL$ is greater than $AD+DF$. And, since $LK=FG$, the perimeter of $\triangle ABC$ is greater than the perimeter of $\triangle ADE$.

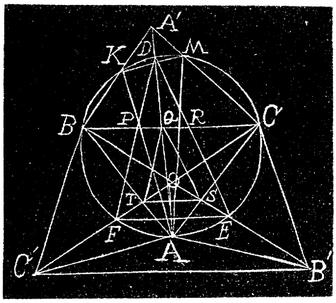
Excellent demonstrations were received from *P. H. PHILBRICK*, *G. B. M. ZERR*, *HENRY HEATON*, *C. A. LINDEMAN*, and *T. T. DAVIS*.

180. Proposed by R. TUCKER, M. A.

ABC is a triangle; A', B', C' are the images of A, B, C with respect to BC, CA, AB. The circum-circle ABC cuts $A'BC$ (say) in K (on $A'B$), M (on $A'C$) and AK, AM, AA' cut BC in P, R, Q, respectively. Prove that (1) the orthocenters of the associated triangles lie on circle ABC; (2) triangle AKM has its sides parallel to and equal twice the sides of the pedal triangle of ABC, and is also equal triangle formed by the above-named orthocenters; (3) $CP.a=b^2$, $BR.a=c^2$, $AP.a=AR.a=bc$, $BP.a=a^2-b^2$, $CR.a=a^2-c^2$, *i. e.*, $PR.a=2bc\cos A$, (4) hence BA touches circle ARC, which contains a Brocard-point of ABC; similarly for CA and circle APB; (5) $BR.CR'$, $AR''=abc=CP.BP'$, AP'' (where R', R'', P', P' correspond to R, P, on CA, AB. respectively); K, K' are the Brocard constants $(k=a^2+b^2+c^2)$ of ABB, $A'B'C'$; then $K'=K=\triangle^2/R^2$.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

(1) Since the triangles $A'BC$, $B'AC$, $C'AB$ are equal to ABC, respectively, and A', B', C' are the images of A, B, C, the orthocenters of the triangles are the images of the orthocenter O, of the triangle ABC with respect to its sides.



But $BS.SE=AS.SC$, or $a\sin C.SE=a\cos C.c\cos A$. $\therefore$ $SE=c\cos A\cot C=SO$.

Similarly, $DQ=b\cos C\cot B=QO$, $TF=a\cos B\cot A=TO$.

$\therefore$ D, E, F are the orthocentres of the triangles.

(2) ArcKDC = arcCEA, both measured by $\angle B$.

ArcDC=arcCE, both measured by $\angle(\frac{1}{2}\pi-C)$.

$\therefore$ ArcKD=arcAE and DE is parallel to KA.

ArcMKB=arcBFA, both measured by $\angle C$.

ArcDB=arcBF, both measured by $\angle(\frac{1}{2}\pi-B)$.

$\therefore$ ArcDM=arcFA, and DF is parallel to AM.

$\therefore$ ArcKBA=arcDBF, arcDCE=arcMEA, arcKDM=arcFAE.

$\therefore$ $DF=KA=2QT$, $DE=MA=2QS$, $KM=FE=2TS$.

Also DE is parallel to AK is parallel to QS, DF is parallel to MA is parallel to QT, FE is parallel to TS. $\triangle DFE=\triangle AKM$ (three sides of one equal three sides of other).

(3) From triangle PAC, $\angle PAC=\angle B$, $\angle P=\angle A$.

$\therefore$ $CP\sin A=b\sin B$ or $CP.a=b^2$.

Similarly, from triangle BAR, $BR\sin A=c\sin C$, or $BR.a=c^2$, $\angle P=\angle R=\angle A$.

$\therefore AP=AR$ and $AP.a=AR.a$.

But $AP\sin A=b\sin C$ or $AP.a=bc=AR.a$.

$BP.a=(a-CP)a=a^2-CP.a=a^2-b^2$.

$CR.a=(a-BR)a=a^2-BR.a=a^2-c^2$.

$PR\sin A=AP\sin(\pi-2A)=2AP\sin A\cos A$.

$\therefore PR.a=2AP.a\cos A=2bc\cos A$.

(4) Since $BR.BC=AB^2$, AB touches the circle through ARC at A: therefore one of the Brocard points is on this circumference. Since $CP.CB=CA^2$, CA touches the circle through APB at A, which contains the other Brocard point.

(5) $BR=c^2/a$, $CR'=a^2/b$, $AR''=b^2/c$, $CP=b^2/a$, $BP'=a^2/c$, $AP''=c^2/b$.

$\therefore BR.CR'.AR''=CP.BP'.AP''=abc$; $(B'C')^2=c^2+b^3-2bc\cos 3A=a^2+8bc\cos A\sin^2 A$; $(A'C')^2=b^2+8ac\cos B\sin^2 B$, $(A'B')^2=c^2+8ab\cos C\sin^2 C$.

$$\therefore K'-K=8(bc\cos A\sin^2 A+ac\cos B\sin^2 B+ab\cos C\sin^2 C)$$
$$=32\triangle^2(\cos A/bc+\cos B/ac+\cos C/ab)$$
$$=(16\triangle^2/a^2b^2c^2)\Sigma(2a^2b^2-a^4)=256\triangle^4/a^2b^2c^2=16\triangle^2/R^2.$$

181. Proposed by **WILLIAM HOOVER, A. M., Ph. D.**, Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

Prove that the extremities of the latera recta of all ellipses having a given major axis $2a$ lie on the parabola $x^2=-a(y-a)$.

Solution by **L. C. WALKER, A. M.**, Petaluma High School, Petaluma, Cal.; **J. R. HITT**, Coral Institute, San Marcos, Tex.; and the **PROPOSER**.

If (x_1, y_1) be an extremity of one of the latera recta, plainly, $y_1=b^2/a$, or $b^2=ay_1$....(1); also, $a^2-b^2=a^2e^2=x_1^2$....(2), b and e having the usual meanings. Eliminating b from (1) and (2), $x_1^2=-a(y_1-a)$.

Also solved by *J. SCHEFFER*, and *G. B. M. ZERR*.

CALCULUS.

137. Proposed by **F. P. MATZ, Sc. D., Ph. D.**, Professor of Mathematics and Astronomy in Defiance College, Defiance, Ohio.

Develop the equation of the curve assumed by the inextensible and revolving skipping rope.

No solution of this problem has been received.

138. Proposed by **M. E. GRABER, A. B.**, Tutor in Mathematics, Heidelberg University, Tiffin, O.

Find the curve the length of whose arc measured from a given point is a mean proportional between the ordinate and twice the abscissa.

Solution by **G. B. M. ZERR, A. M., Ph. D.**, Professor of Chemistry and Physics in The Temple College, Philadelphia, Pa., and the **PROPOSER**.

From the problem, $s^2=2xy$ or $s=\sqrt{(2xy)}$.

$$ds=\sqrt{(dx^2+dy^2)}=\frac{1}{\sqrt{2}}[\sqrt{(y/x)}dx+\sqrt{(x/y)}dy] \text{ or}$$